

## Problem 2.20

[Difficulty: 3]

**2.20** Consider the flow described by the velocity field  $\vec{V} = Bx(1 + At)\hat{i} + Cy\hat{j}$ , with  $A = 0.5 \text{ s}^{-1}$  and  $B = C = 1 \text{ s}^{-1}$ . Coordinates are measured in meters. Plot the pathline traced out by the particle that passes through the point (1, 1) at time  $t = 0$ . Compare with the streamlines plotted through the same point at the instants  $t = 0, 1$ , and  $2 \text{ s}$ .

**Given:** Velocity field

**Find:** Plot of pathline traced out by particle that passes through point (1,1) at  $t = 0$ ; compare to streamlines through same point at the instants  $t = 0, 1$  and  $2 \text{ s}$

**Solution:**

**Governing equations:** For pathlines  $u_p = \frac{dx}{dt}$   $v_p = \frac{dy}{dt}$  For streamlines  $\frac{v}{u} = \frac{dy}{dx}$

**Assumption:** 2D flow

Hence for pathlines  $u_p = \frac{dx}{dt} = B \cdot x \cdot (1 + A \cdot t)$   $A = 0.5 \cdot \frac{1}{s}$   $B = 1 \cdot \frac{1}{s}$   $v_p = \frac{dy}{dt} = C \cdot y$   $C = 1 \cdot \frac{1}{s}$

So, separating variables  $\frac{dx}{x} = B \cdot (1 + A \cdot t) \cdot dt$   $\frac{dy}{y} = C \cdot dt$

Integrating  $\ln(x) = B \cdot \left( t + A \cdot \frac{t^2}{2} \right) + C_1$   $\ln(y) = C \cdot t + C_2$

$$x = e^{B \cdot \left( t + A \cdot \frac{t^2}{2} \right) + C_1} = e^{C_1} \cdot e^{B \cdot \left( t + A \cdot \frac{t^2}{2} \right)} = c_1 \cdot e^{B \cdot \left( t + A \cdot \frac{t^2}{2} \right)}$$

$$y = e^{C \cdot t + C_2} = e^{C_2} \cdot e^{C \cdot t} = c_2 \cdot e^{C \cdot t}$$

The pathlines are  $x = c_1 \cdot e^{B \cdot \left( t + A \cdot \frac{t^2}{2} \right)}$   $y = c_2 \cdot e^{C \cdot t}$

Using given data  $x = e^{B \cdot \left( t + A \cdot \frac{t^2}{2} \right)}$   $y = e^{C \cdot t}$

For streamlines  $\frac{v}{u} = \frac{dy}{dx} = \frac{C \cdot y}{B \cdot x \cdot (1 + A \cdot t)}$

So, separating variables  $(1 + A \cdot t) \cdot \frac{dy}{y} = \frac{C}{B} \cdot \frac{dx}{x}$  which we can integrate for any given  $t$  ( $t$  is treated as a constant)

Integrating  $(1 + A \cdot t) \cdot \ln(y) = \frac{C}{B} \cdot \ln(x) + c$

The solution is  $y^{1+A \cdot t} = \text{const} \cdot x^{\frac{C}{B}}$  or  $y = \text{const} \cdot x$

For particles at (1,1) at t = 0, 1, and 2s  $y = x^{\frac{C}{B}}$   $y = x^{\frac{C}{(1+A)B}}$   $y = x^{\frac{C}{(1+2 \cdot A)B}}$

